PROBLEM SOLUTIONS

9.1 Three variables that determine the microstructure of an alloy are 1) the alloying elements present, 2) the concentrations of these alloying elements, and 3) the heat treatment of the alloy.

9.2 In order for a system to exist in a state of equilibrium the free energy must be a minimum for some specified combination of temperature, pressure, and composition.

9.3 Diffusion occurs during the development of microstructure in the absence of a concentration gradient because the driving force is different than for steady state diffusion as described in Section 5.3; for the development of microstructure, the driving force is a decrease in free energy.

9.4 For the condition of phase equilibrium the free energy is a minimum, the system is completely stable meaning that over time the phase characteristics are constant. For metastability, the system is not at equilibrium, and there are very slight (and often imperceptible) changes of the phase characteristics with time.

9.5 This problem asks that we cite the phase or phases present for several alloys at specified temperatures.
   (a) For an alloy composed of 90 wt% Zn-10 wt% Cu and at 400°C, from Figure 9.17, ε and η phases are present, and
       \[ C_\varepsilon = 87 \text{ wt}\% \text{ Zn-13 wt}\% \text{ Cu} \]
       \[ C_\eta = 97 \text{ wt}\% \text{ Zn-3 wt}\% \text{ Cu} \]

   (b) For an alloy composed of 75 wt% Sn-25 wt% Pb and at 175°C, from Figure 9.7, α and β phases are present, and
       \[ C_\alpha = 15 \text{ wt}\% \text{ Sn-85 wt}\% \text{ Pb} \]
       \[ C_\beta = 98 \text{ wt}\% \text{ Sn-2 wt}\% \text{ Pb} \]
(c) For an alloy composed of 55 wt% Ag-45 wt% Cu and at 900°C, from Figure 9.6, only the liquid phase is present; its composition is 55 wt% Ag-45 wt% Cu.

(d) For an alloy composed of 30 wt% Pb-70 wt% Mg and at 425°C, from Figure 9.18, only the α phase is present; its composition is 30 wt% Pb-70 wt% Mg.

(e) For an alloy composed of 2.12 kg Zn and 1.88 kg Cu and at 500°C, we must first determine the Zn and Cu concentrations, as

\[
C_{\text{Zn}} = \frac{2.12 \text{ kg}}{2.12 \text{ kg} + 1.88 \text{ kg}} \times 100 = 53 \text{ wt%}
\]

\[
C_{\text{Cu}} = \frac{1.88 \text{ kg}}{2.12 \text{ kg} + 1.88 \text{ kg}} \times 100 = 47 \text{ wt%}
\]

From Figure 9.17, \(\beta + \gamma\) phases are present, and

\[
C_{\beta} = 49 \text{ wt% Zn-51 wt% Cu}
\]

\[
C_{\gamma} = 57 \text{ wt% Zn-43 wt% Cu}
\]

(f) For an alloy composed of 37 lb\text{ m} Pb and 6.5 lb\text{ m} Mg and at 400°C, we must first determine the Pb and Mg concentrations, as

\[
C_{\text{Pb}} = \frac{37 \text{ lb}_{\text{m}}}{37 \text{ lb}_{\text{m}} + 6.5 \text{ lb}_{\text{m}}} \times 100 = 85 \text{ wt%}
\]

\[
C_{\text{Mg}} = \frac{6.5 \text{ lb}_{\text{m}}}{37 \text{ lb}_{\text{m}} + 6.5 \text{ lb}_{\text{m}}} \times 100 = 15 \text{ wt%}
\]

From Figure 9.18, Mg\textsubscript{2}Pb and L phases are present, and

\[
C_{\text{Mg}_2\text{Pb}} = 81 \text{ wt% Pb-19 wt% Mg}
\]

\[
C_{\text{L}} = 93 \text{ wt% Pb-7 wt% Mg}
\]

(g) For an alloy composed of 8.2 mol Ni and 4.3 mol Cu and at 1250°C, it is first necessary to determine the Ni and Cu concentrations, which we will do in wt% as follows:
\[ m_{Ni}' = n_{Ni}A_{Ni} = (8.2 \text{ mol})(58.69 \text{ g/mol}) = 481.3 \text{ g} \]

\[ m_{Cu}' = n_{Cu}A_{Cu} = (4.3 \text{ mol})(63.55 \text{ g/mol}) = 273.3 \text{ g} \]

\[ C_{Ni} = \frac{481.3 \text{ g}}{481.3 \text{ g} + 273.3 \text{ g}} \times 100 = 63.8 \text{ wt\%} \]

\[ C_{Cu} = 100 \text{ wt\%} - 63.8 \text{ wt\%} = 36.2 \text{ wt\%} \]

From Figure 9.2a, only the \( \alpha \) phase is present; its composition is 63.8 wt\% Ni-36.2 wt\% Cu.

(h) For an alloy composed of 4.5 mol Sn and 0.45 mol Pb and at 200°C, it is first necessary to determine the Sn and Pb concentrations, which we will do in weight percent as follows:

\[ m_{Sn}' = n_{Sn}A_{Sn} = (4.5 \text{ mol})(118.69 \text{ g/mol}) = 534.1 \text{ g} \]

\[ m_{Pb}' = n_{Pb}A_{Pb} = (0.45 \text{ mol})(207.2 \text{ g/mol}) = 93.2 \text{ g} \]

\[ C_{Sn} = \frac{534.1 \text{ g}}{534.1 \text{ g} + 93.2 \text{ g}} \times 100 = 85 \text{ wt\%} \]

\[ C_{Pb} = \frac{93.2 \text{ g}}{534.1 \text{ g} + 93.2 \text{ g}} \times 100 = 15 \text{ wt\%} \]

From Figure 9.7, \( \beta \) and liquid phases are present; and

\[ C_{\beta} = 98 \text{ wt\% Sn-2 w\% Pb} \]
\[ C_{L} = 74 \text{ wt\% Sn-26 wt\% Pb} \]

9.6 This problem asks us to determine the phases present and their concentrations at several temperatures, as an alloy of composition 74 wt\% Zn-26 wt\% Cu is cooled. From Figure 9.17:

At 850°C, a liquid phase is present; \( C_{L} = 74 \text{ wt\% Zn-26 wt\% Cu} \)

At 750°C, \( \gamma \) and liquid phases are present; \( C_{\gamma} = 76 \text{ wt\% Zn-24 wt\% Cu}; C_{L} = 68 \text{ wt\% Zn-32 wt\% Cu} \)
At 680°C, δ and liquid phases are present; \( C_\delta = 74 \text{ wt\% Zn-26 wt\% Cu} \); \( C_L = 82 \text{ wt\% Zn-18 wt\% Cu} \)

At 600°C, the δ phase is present; \( C_\delta = 74 \text{ wt\% Zn-26 wt\% Cu} \)

At 500°C, γ and ε phases are present; \( C_\gamma = 69 \text{ wt\% Zn-31 wt\% Cu} \); \( C_\varepsilon = 78 \text{ wt\% Zn-22 wt\% Cu} \)

9.7 This problem asks that we determine the phase mass fractions for the alloys and temperatures in Problem 9.5.

(a) \[
W_\varepsilon = \frac{C_\eta - C_\phi}{C_\eta - C_\varepsilon} = \frac{97 - 90}{97 - 87} = 0.70
\]

\[
W_\eta = \frac{C_\phi - C_\varepsilon}{C_\eta - C_\varepsilon} = \frac{90 - 87}{97 - 87} = 0.30
\]

(b) \[
W_\alpha = \frac{C_\beta - C_\phi}{C_\beta - C_\alpha} = \frac{98 - 75}{98 - 15} = 0.28
\]

\[
W_\beta = \frac{C_\phi - C_\alpha}{C_\beta - C_\alpha} = \frac{75 - 15}{98 - 15} = 0.72
\]

(c) \( W_L = 1.0 \)

(d) \( W_\alpha = 1.0 \)

(e) \[
W_\beta = \frac{C_\gamma - C_\phi}{C_\gamma - C_\beta} = \frac{57 - 53}{57 - 49} = 0.50
\]

\[
W_\gamma = \frac{C_\phi - C_\beta}{C_\gamma - C_\beta} = \frac{53 - 49}{57 - 49} = 0.50
\]
\[ W_{\text{Mg}_2\text{Pb}} = \frac{C_L - C_0}{C_L - C_{\text{Mg}_2\text{Pb}}} = \frac{93 - 85}{93 - 81} = 0.67 \]

\[ W_L = \frac{C_0 - C_{\text{Mg}_2\text{Pb}}}{C_L - C_{\text{Mg}_2\text{Pb}}} = \frac{85 - 81}{93 - 81} = 0.33 \]

\( W_\alpha = 1.0 \)

\[ W_\beta = \frac{C_0 - C_L}{C_\beta - C_L} = \frac{85 - 74}{98 - 74} = 0.46 \]

\[ W_L = \frac{C_\beta - C_0}{C_\beta - C_L} = \frac{98 - 85}{98 - 74} = 0.54 \]

9.8 (a) In this problem we are asked to derive Equation (9.6a), which is used to convert from phase weight fraction to phase volume fraction. Volume fraction of phase \( \alpha \), \( V_\alpha \), is defined by Equation (9.5) as

\[ V_\alpha = \frac{v_\alpha}{v_\alpha + v_\beta} \]  

(9.51)

where \( v_\alpha \) and \( v_\beta \) are the volumes of the respective phases in the alloy. Furthermore, the density of each phase is equal to the ratio of its mass and volume, or upon rearrangement

\[ v_\alpha = \frac{m_\alpha}{\rho_\alpha} \]  

(9.52a)

\[ v_\beta = \frac{m_\beta}{\rho_\beta} \]  

(9.52b)

Substitution of these expressions into Equation (9.51) leads to

\[ V_\alpha = \frac{\frac{m_\alpha}{\rho_\alpha}}{\frac{m_\alpha}{\rho_\alpha} + \frac{m_\beta}{\rho_\beta}} \]  

(9.53)
in which \(m\)'s and \(\rho\)'s denote masses and densities, respectively. Now, the mass fractions of the \(\alpha\) and \(\beta\) phases (i.e., \(W_\alpha\) and \(W_\beta\)) are defined in terms of the phase masses as

\[
W_\alpha = \frac{m_\alpha}{m_\alpha + m_\beta} \tag{9.54a}
\]

\[
W_\beta = \frac{m_\beta}{m_\alpha + m_\beta} \tag{9.54b}
\]

Which, upon rearrangement yield

\[
m_\alpha = W_\alpha (m_\alpha + m_\beta) \tag{9.55a}
\]

\[
m_\beta = W_\beta (m_\alpha + m_\beta) \tag{9.55b}
\]

Incorporation of these relationships into Equation (9.53) leads to the desired expression

\[
V_\alpha = \frac{W_\alpha}{\rho_\alpha} \tag{9.56}
\]

(b) For this portion of the problem we are asked to derive Equation (9.7a), which is used to convert from phase volume fraction to mass fraction. Mass fraction of the \(\alpha\) phase is defined as

\[
W_\alpha = \frac{m_\alpha}{m_\alpha + m_\beta} \tag{9.7}
\]

From Equations (9.52a) and (9.52b)

\[
m_\alpha = v_\alpha \rho_\alpha \tag{9.8a}
\]

\[
m_\beta = v_\beta \rho_\beta \tag{9.8b}
\]

Substitution of these expressions into Equation (9.7) yields

\[
W_\alpha = \frac{v_\alpha \rho_\alpha}{v_\alpha \rho_\alpha + v_\beta \rho_\beta} \tag{9.9}
\]
From Equation (9.91) and its equivalent for $V_\beta$, the following may be written

$$v_\alpha = V_\alpha (v_\alpha + v_\beta) \quad (9.91a)$$

$$v_\beta = V_\beta (v_\alpha + v_\beta) \quad (9.91b)$$

Substitution of Equations (9.91a) and (9.91b) into Equation (9.99) yields the desired expression

$$w_\alpha = \frac{V_\alpha \rho_\alpha}{V_\alpha \rho_\alpha + V_\beta \rho_\beta} \quad (9.91)$$

9.9 This problem asks that we determine the phase volume fractions for the alloys and temperatures in Problem 9.5a, b, and c. This is accomplished by using the technique illustrated in Example Problem 9.3, and the results of Problem 9.7.

(a) This is a Cu-Zn alloy at 400°C, wherein

- $C_\epsilon = 87$ wt% Zn-13 wt% Cu
- $C_\eta = 97$ wt% Zn-3 wt% Cu
- $W_\epsilon = 0.70$
- $W_\eta = 0.30$
- $\rho_{Cu} = 8.77 \text{ g/cm}^3$
- $\rho_{Zn} = 6.83 \text{ g/cm}^3$

Using this data it is first necessary to compute the densities of the $\epsilon$ and $\eta$ phases using Equation (4.10a). Thus

$$\rho_\epsilon = \frac{100}{C_{Zn(\epsilon)} \rho_{Zn} + C_{Cu(\epsilon)} \rho_{Cu}}$$

$$= \frac{87}{6.83 \text{ g/cm}^3} + \frac{13}{8.77 \text{ g/cm}^3} = 7.03 \text{ g/cm}^3$$
\[ \rho_\eta = \frac{100}{\frac{C_{\text{Zn}}(n)}{\rho_\text{Zn}} + \frac{C_{\text{Cu}}(n)}{\rho_\text{Cu}}} \]

\[ = \frac{\frac{100}{97}}{\frac{6.83 \text{ g/cm}^3}{3} + \frac{8.77 \text{ g/cm}^3}{8.77 \text{ g/cm}^3}} = 6.88 \text{ g/cm}^3 \]

Now we may determine the \( V_\varepsilon \) and \( V_\eta \) values using Equation 9.6. Thus,

\[ V_\varepsilon = \frac{W_\varepsilon}{\frac{\rho_\varepsilon}{\rho_\varepsilon} + \frac{W_\eta}{\rho_\eta}} \]

\[ = \frac{0.70}{7.03 \text{ g/cm}^3} \frac{1}{0.70 + 0.30} = 0.70 \]

\[ V_\eta = \frac{W_\eta}{\frac{\rho_\eta}{\rho_\varepsilon} + \frac{W_\eta}{\rho_\eta}} \]

\[ = \frac{0.30}{6.88 \text{ g/cm}^3} \frac{1}{0.70 + 0.30} = 0.30 \]

(b) This is a Pb-Sn alloy at 175°C, wherein

\[ C_\alpha = 15 \text{ wt}\% \text{ Sn-85 wt}\% \text{ Pb} \]
\[ C_\beta = 98 \text{ wt}\% \text{ Sn-2 wt}\% \text{ Pb} \]
\[ W_\alpha = 0.28 \]
\[ W_\beta = 0.72 \]
\[ \rho_{\text{Sn}} = 7.22 \text{ g/cm}^3 \]
\[ \rho_{\text{Pb}} = 11.20 \text{ g/cm}^3 \]

Using this data it is first necessary to compute the densities of the \( \alpha \) and \( \beta \) phases. Thus
Now we may determine the $V_\alpha$ and $V_\beta$ values using Equation (9.6). Thus,

\[
V_\alpha = \frac{W_\alpha \rho_\alpha}{W_\alpha \rho_\alpha + W_\beta \rho_\beta} = \frac{0.28 \cdot 10.34 \text{ g/cm}^3}{0.28 \cdot 10.34 \text{ g/cm}^3 + 0.72 \cdot 7.27 \text{ g/cm}^3} = 0.21
\]

\[
V_\beta = \frac{W_\beta \rho_\beta}{W_\alpha \rho_\alpha + W_\beta \rho_\beta} = \frac{0.72 \cdot 7.27 \text{ g/cm}^3}{0.28 \cdot 10.34 \text{ g/cm}^3 + 0.72 \cdot 7.27 \text{ g/cm}^3} = 0.79
\]

(c) This is a Ag-Cu alloy at 900°C, wherein only the liquid phase is present. Therefore, $V_L = 1.0$. 

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9.10 (a) Spreading salt on ice will lower the melting temperature, since the liquidus line decreases from 0°C to the eutectic temperature at about -21°C. Thus, ice at a temperature below 0°C (and above -21°C) can be made to form a liquid phase by the addition of salt.
(b) We are asked to compute the concentration of salt necessary to have a 50% ice-50% brine solution at -10°C (14°F). At -10°C,

\[
C_{\text{ice}} = 0 \text{ wt\% NaCl-100 wt\% H}_2\text{O}
\]

\[
C_{\text{brine}} = 13 \text{ wt\% NaCl-87 wt\% H}_2\text{O}
\]

Thus,

\[
W_{\text{ice}} = 0.5 = \frac{C_{\text{brine}} - C_0}{C_{\text{brine}} - C_{\text{ice}}} = \frac{13 - C_0}{13 - 0}
\]

Solving for \(C_0\) (the concentration of salt) yields a value of 6.5 wt\% NaCl-93.5 wt\% H\(_2\)O.

9.11 (a) This part of the problem calls for us to cite the temperature to which a 90 wt\% Pb-10 wt\% Sn alloy must be heated in order to have 50% liquid. Probably the easiest way to solve this problem is by trial and error—that is, moving vertically at the given composition, through the \(\alpha + L\) region until the tie-line lengths on both sides of the given composition are the same (Figure 9.7). This occurs at approximately 300°C (570°F).
(b) We can also produce a 50% liquid solution at 250°C, by adding Sn to the alloy. At 250°C and within the \(\alpha + L\) phase region

\[
C_{\alpha} = 13 \text{ wt\% Sn-87 wt\% Pb}
\]

\[
C_L = 39 \text{ wt\% Sn-61 wt\% Pb}
\]

Let \(C_0\) be the new alloy composition to give \(W_{\alpha} = W_L = 0.5\). Then,

\[
W_{\alpha} = 0.5 = \frac{C_L - C_0}{C_L - C_{\alpha}} = \frac{39 - C_0}{39 - 13}
\]

And solving for \(C_0\) gives 26 wt\% Sn. Now, let \(m_{\text{Sn}}\) be the mass of Sn added to the alloy to achieve this new composition. The amount of Sn in the original alloy is

\[
(0.10)(1.5 \text{ kg}) = 0.15 \text{ kg}
\]
Then, using a modified form of Equation (4.3)

\[
\begin{bmatrix}
0.15 \text{ kg} + m_{Sn} \\
1.5 \text{ kg} + m_{Sn}
\end{bmatrix} \times 100 = 26
\]

And, solving for \(m_{Sn}\) yields \(m_{Sn} = 0.324\) kg.

9.12 (a) We are asked to determine how much sugar will dissolve in 1500 g of water at 90°C. From the solubility limit curve in Figure 9.1, at 90°C the maximum concentration of sugar in the syrup is about 77 wt%. It is now possible to calculate the mass of sugar using Equation (4.3) as

\[
C_{\text{sugar (wt\%)}} = \frac{m_{\text{sugar}}}{m_{\text{sugar}} + m_{\text{water}}} \times 100
\]

\[
77 \text{ wt\%} = \frac{m_{\text{sugar}}}{m_{\text{sugar}} + 1500 \text{ g}} \times 100
\]

Solving for \(m_{\text{sugar}}\) yields \(m_{\text{sugar}} = 5022\) g

(b) Again using this same plot, at 20°C the solubility limit (or the concentration of the saturated solution) is about 64 wt% sugar.

(c) The mass of sugar in this saturated solution at 20°C \((m_{\text{sugar}'}\)\) may also be calculated using Equation (4.3) as follows:

\[
64 \text{ wt\%} = \frac{m_{\text{sugar}'}}{m_{\text{sugar}'} + 1500 \text{ g}} \times 100
\]

which yields a value for \(m_{\text{sugar}'}\) of 2667 g. Subtracting the latter from the former of these sugar concentrations yields the amount of sugar that precipitated out of the solution upon cooling \(m_{\text{sugar}''}\); that is

\[
m_{\text{sugar}''} = m_{\text{sugar}} - m_{\text{sugar}'} = 5022 \text{ g} - 2667 \text{ g} = 2355 \text{ g}
\]

9.13 This problem asks us to consider a specimen of ice \(I\) which is at -10°C and 1 atm pressure.

(a) In order to determine the pressure at which melting occurs at this temperature, we move vertically at this temperature until we cross the Ice \(I\)-Liquid phase boundary of Figure 9.34. This
occurs at approximately 570 atm; thus the pressure of the specimen must be raised from 1 to 570 atm.

(b) In order to determine the pressure at which sublimation occurs at this temperature, we move vertically downward from 1 atm until we cross the Ice I-Vapor phase boundary of Figure 9.34. This intersection occurs at approximately 0.0023 atm.

9.14 The melting and boiling temperatures for ice I at a pressure of 0.01 atm may be determined by moving horizontally across the pressure-temperature diagram of Figure 9.34 at this pressure. The temperature corresponding to the intersection of the Ice I-Liquid phase boundary is the melting temperature, which is approximately 1°C. On the other hand, the boiling temperature is at the intersection of the horizontal line with the Liquid-Vapor phase boundary—approximately 28°C.

9.15 (a) This portion of the problem asks that we calculate, for a Pb-Mg alloy, the mass of lead in 5.5 kg of the solid $\alpha$ phase at 200°C just below the solubility limit. From Figure 9.18, the composition of an alloy at this temperature is about 5 wt% Pb. Therefore, the mass of Pb in the alloy is just $(0.05)(5.5 \text{ kg}) = 0.28 \text{ kg}$.

(b) At 350°C, the solubility limit of the $\alpha$ phase increases to approximately 25 wt% Pb. In order to determine the additional amount of Pb that may be added ($m_{\text{Pb}}$), we utilize a modified form of Equation (4.3) as

\[
C_{\text{Pb}} = 25 \text{ wt\%} = \frac{0.28 \text{ kg} + m_{\text{Pb}}}{5.5 \text{ kg} + m_{\text{Pb}}} \times 100
\]

Solving for $m_{\text{Pb}}$ yields $m_{\text{Pb}} = 1.46 \text{ kg}$.

9.16 (a) Coring is the phenomenon whereby concentration gradients exist across grains in polycrystalline alloys, with higher concentrations of the component having the lower melting temperature at the grain boundaries. It occurs, during solidification, as a consequence of cooling rates that are too rapid to allow for the maintenance of the equilibrium composition of the solid phase.

(b) One undesirable consequence of a cored structure is that, upon heating, the grain boundary regions will melt first and at a temperature below the equilibrium phase boundary from the phase diagram; this melting results in a loss in mechanical integrity of the alloy.
9.17 This problem asks if a noncold-worked Cu-Ni solid solution alloy is possible having a minimum tensile strength of 350 MPa (50,750 psi) and also a ductility of at least 48%EL. From Figure 9.5a, a tensile strength greater than 350 MPa is possible for compositions between about 22.5 and 98 wt% Ni. On the other hand, according to Figure 9.5b, ductilities greater than 48%EL exist for compositions less than about 8 wt% and greater than about 98 wt% Ni. Therefore, the stipulated criteria are met only at a composition of 98 wt% Ni.

9.18 It is possible to have a Cu-Ag alloy, which at equilibrium consists of a β phase of composition 92 wt% Ag-8 wt% Cu and a liquid phase of composition 76 wt% Ag-24 wt% Cu. From Figure 9.6 a horizontal tie line can be constructed across the β + L phase region at about 800°C which intersects the L-(β + L) phase boundary at 76 wt% Ag, and also the (β + L)-β phase boundary at 92 wt% Ag.

9.19 It is not possible to have a Cu-Zn alloy, which at equilibrium consists of an ε phase of composition 80 wt% Zn-20 wt% Cu and also a liquid phase of composition 95 wt% Zn-5 wt% Cu. From Figure 9.17 a single tie line does not exist within the ε + L region which intersects the phase boundaries at the given compositions. At 80 wt% Zn, the ε-(ε + L) phase boundary is at about 575°C, whereas at 95 wt% Zn the (ε + L)-L phase boundary is at about 490°C.

9.20 Upon heating a copper-nickel alloy of composition 70 wt% Ni-30 wt% Cu from 1300°C and utilizing Figure 9.2a:
(a) The first liquid forms at the temperature at which a vertical line at this composition intersects the α-(α + L) phase boundary--i.e., about 1350°C;
(b) The composition of this liquid phase corresponds to the intersection with the (α + L)-L phase boundary, of a tie line constructed across the α + L phase region at 1350°C--i.e., 59 wt% Ni;
(c) Complete melting of the alloy occurs at the intersection of this same vertical line at 70 wt% Ni with the (α + L)-L phase boundary--i.e., about 1380°C;
(d) The composition of the last solid remaining prior to complete melting corresponds to the intersection with α-(α + L) phase boundary, of the tie line constructed across the α + L phase region at 1380°C--i.e., about 78 wt% Ni.

9.21 Upon cooling a 50 wt% Pb-50 wt% Mg alloy from 700°C and utilizing Figure 9.18:
(a) The first solid phase forms at the temperature at which a vertical line at this composition intersects the L-(α + L) phase boundary--i.e., about 550°C;
(b) The composition of this solid phase corresponds to the intersection with the $\alpha-(\alpha + L)$ phase boundary, of a tie line constructed across the $\alpha + L$ phase region at 550°C—i.e., 22 wt% Pb-78 wt% Mg;

(c) Complete solidification of the alloy occurs at the intersection of this same vertical line at 50 wt% Pb with the eutectic isotherm—i.e., about 465°C;

(d) The composition of the last liquid phase remaining prior to complete solidification corresponds to the eutectic composition—i.e., about 66 wt% Pb-34 wt% Mg.

9.22 (a) In order to determine the temperature of a 90 wt% Ag-10 wt% Cu alloy for which $\beta$ and liquid phases are present with the liquid phase of composition 85 wt% Ag, we need to construct a tie line across the $\beta + L$ phase region of Figure 9.6 that intersects the liquidus line at 85 wt% Ag; this is possible at about 850°C.

(b) The composition of the $\beta$ phase at this temperature is determined from the intersection of this same tie line with solidus line, which corresponds to about 95 wt% Ag.

(c) The mass fractions of the two phases are determined using the lever rule, Equations (9.1) and (9.2) with $C_o = 90$ wt% Ag, $C_L = 85$ wt% Ag, and $C_\beta = 95$ wt% Ag, as

$$W_\beta = \frac{C_o \cdot C_L}{C_\beta \cdot C_L} = \frac{90 \cdot 85}{95 \cdot 85} = 0.50$$

$$W_L = \frac{C_\beta \cdot C_o}{C_\beta \cdot C_L} = \frac{95 \cdot 90}{95 \cdot 85} = 0.50$$

9.23 The germanium-silicon phase diagram is constructed below.
9.24 (a) We are given that the mass fractions of $\alpha$ and liquid phases are both 0.5 for a 30 wt% Sn-70 wt% Pb alloy and asked to estimate the temperature of the alloy. Using the appropriate phase diagram, Figure 9.7, by trial and error with a ruler, a tie line within the $\alpha + L$ phase region that is divided in half for an alloy of this composition exists at about 230°C.

(b) We are now asked to determine the compositions of the two phases. This is accomplished by noting the intersections of this tie line with both the solidus and liquidus lines. From these intersections, $C_\alpha = 15$ wt% Sn, and $C_L = 42$ wt% Sn.

9.25 The problem is to solve for compositions at the phase boundaries for both $\alpha$ and $\beta$ phases (i.e., $C_\alpha$ and $C_\beta$). We may set up two independent lever rule expressions, one for each composition, in terms of $C_\alpha$ and $C_\beta$ as follows:

$$W_{\alpha 1} = 0.57 = \frac{C_\beta - C_{\alpha 1}}{C_\beta - C_\alpha} = \frac{C_\beta - 60}{C_\beta - C_\alpha}$$

$$W_{\alpha 2} = 0.14 = \frac{C_\beta - C_{\alpha 2}}{C_\beta - C_\alpha} = \frac{C_\beta - 30}{C_\beta - C_\alpha}$$

In these expressions, compositions are given in wt% A. Solving for $C_\alpha$ and $C_\beta$ from these equations, yield

$$C_\alpha = 90 \text{ (or 90 wt% A-10 wt% B)}$$
\[ C_\beta = 20.2 \text{ (or 20.2 wt\% A-79.8 wt\% B)} \]

9.26 For this problem

\[ C_\alpha = 55 \text{ (or 55 wt\% B-45 wt\% A)} \]
\[ C_\beta = 90 \text{ (or 90 wt\% B-10 wt\% A)} \]

\[ W_\alpha = W_\beta = 0.5 \]

If we set up the lever rule for \( W_\alpha \)

\[ W_\alpha = 0.5 = \frac{C_\beta - C_\alpha}{C_\beta - C_\alpha} = \frac{90 - 55}{90 - C_\alpha} \]

And solving for \( C_\alpha \)

\[ C_\alpha = 20 \text{ (or 20 wt\% B-80 wt\% A)} \]

9.27 It is not possible to have a Cu-Ag alloy of composition 50 wt\% Ag-50 wt\% Cu which consists of mass fractions \( W_\alpha = 0.60 \) and \( W_\beta = 0.40 \). Using the appropriate phase diagram, Figure 9.6, and, using Equations (9.1) and (9.2) let us determine \( W_\alpha \) and \( W_\beta \) at just below the eutectic temperature and also at room temperature. At just below the eutectic, \( C_\alpha = 8.0 \text{ wt\% Ag} \) and \( C_\beta = 91.2 \text{ wt\% Ag} \); thus,

\[ W_\alpha = \frac{C_\beta - C_\alpha}{C_\beta - C_\alpha} = \frac{91.2 - 50}{91.2 - 8} = 0.50 \]
\[ W_\beta = 1.0 - W_\alpha = 1.0 - 0.5 = 0.50 \]

Furthermore, at room temperature, \( C_\alpha = 0 \text{ wt\% Ag} \) and \( C_\beta = 100 \text{ wt\% Ag} \); employment of Equations (9.1) and (9.2) yields

\[ W_\alpha = \frac{C_\beta - C_\alpha}{C_\beta - C_\alpha} = \frac{100 - 50}{100 - 0} = 0.50 \]
And, $W_\beta = 0.50$. Thus, the mass fractions of the $\alpha$ and $\beta$ phases, upon cooling through the $\alpha + \beta$ phase region will remain approximately constant at about 0.5, and will never have values of $W_\alpha = 0.60$ and $W_\beta = 0.40$ as called for in the problem.

9.28 Yes, it is possible to have a 30 wt% Pb-70 wt% Mg alloy which has masses of 7.39 kg and 3.81 kg for the $\alpha$ and Mg$_2$Pb phases, respectively. In order to demonstrate this, it is first necessary to determine the mass fraction of each phase as follows:

$$W_\alpha = \frac{m_\alpha}{m_\alpha + m_{Mg_2Pb}} = \frac{7.39\ kg}{7.39\ kg + 3.81\ kg} = 0.66$$

$$W_{Mg_2Pb} = 1.00 - 0.66 = 0.34$$

Now, if we apply the lever rule expression for $W_\alpha$:

$$W_\alpha = \frac{C_{Mg_2Pb} - C_\alpha}{C_{Mg_2Pb} - C_o}$$

Since the Mg$_2$Pb phase exists only at 81 wt% Pb, and $C_o = 30$ wt% Pb

$$W_\alpha = 0.66 = \frac{81 - 30}{81 - C_\alpha}$$

Solving for $C_\alpha$ from this expression yields $C_\alpha = 3.7$ wt% Pb. The position along the $\alpha-(\alpha + Mg_2Pb)$ phase boundary of Figure 9.18 corresponding to this composition is approximately 190°C.

9.29 (a) From Figure 9.6, the maximum solubility of Cu in Ag at 700°C corresponds to the position of the $\beta-(\alpha + \beta)$ phase boundary at this temperature, or to about 6 wt% Cu.

(b) From this same figure, the maximum solubility of Ag in Cu corresponds to the position of the $\alpha-(\alpha + \beta)$ phase boundary at this temperature, or about 5 wt% Ag.

9.30 We are asked to determine the approximate temperature from which a Pb-Mg alloy was quenched, given the mass fractions of $\alpha$ and Mg$_2$Pb phases. We can write a lever-rule expression for the mass fraction of the $\alpha$ phase as
The value of $C_o$ is stated as 45 wt% Pb-55 wt% Mg, and $C_{Mg2Pb}$ is 81 wt% Pb-19 wt% Mg, which is independent of temperature (Figure 9.18); thus,

$$0.65 = \frac{81 - 45}{81 - C_\alpha}$$

which yields

$$C_\alpha = 25.6 \text{ wt% Pb}$$

The temperature at which the $\alpha$-(Mg$_2$Pb) phase boundary (Figure 9.18) has a value of 25.6 wt% Pb is about 360°C (680°F).

9.31 This problem asks if it is possible to have a Cu-Ag alloy for which the mass fractions of primary $\beta$ and total $\beta$ are 0.68 and 0.925, respectively at 775°C. In order to make this determination we need to set up the appropriate lever rule expression for each of these quantities. From Figure 9.6 and at 775°C, $C_\alpha = 8.0 \text{ wt% Ag}$, $C_\beta = 91.2 \text{ wt% Ag}$, and $C_{eutectic} = 71.9 \text{ wt% Ag}$

For primary $\beta$

$$W_\beta = \frac{C_o \cdot C_{eutectic}}{C_\beta \cdot C_{eutectic}} = \frac{C_o \cdot 71.9}{91.2 \cdot 71.9} = 0.68$$

Solving for $C_o$ gives $C_o = 85 \text{ wt% Ag}$.

Now the analogous expression for total $\beta$

$$W_\beta = \frac{C_o \cdot C_\alpha}{C_\beta \cdot C_\alpha} = \frac{C_o \cdot 8.0}{91.2 \cdot 8.0} = 0.925$$

And this value of $C_o$ is 85 wt% Ag. Therefore, since these two $C_o$ values are the same, this alloy is possible.

9.32 This problem asks if it is possible to have a Mg-Pb alloy for which the masses of primary $\alpha$ and total $\alpha$ are 4.23 kg and 6.00 kg, respectively in 6.7 kg total of the alloy at 460°C. In order to make this determination we first need to convert these masses to mass fractions. Thus,
\[ W_{\alpha'} = \frac{4.23 \text{ kg}}{6.7 \text{ kg}} = 0.631 \]

\[ W_{\alpha} = \frac{6.00 \text{ kg}}{6.7 \text{ kg}} = 0.896 \]

Next it is necessary to set up the appropriate lever rule expression for each of these quantities. From Figure 9.18 and at 460°C, \( c_\alpha = 41 \text{ wt}\% \text{ Pb} \), \( c_{\text{Mg}_2\text{Pb}} = 81 \text{ wt}\% \text{ Pb} \), and \( c_{\text{eutectic}} = 66 \text{ wt}\% \text{ Pb} \).

For primary \( \alpha \)

\[ W_{\alpha'} = \frac{c_{\text{eutectic}} - c_0}{c_{\text{eutectic}} - c_\alpha} \]
\[ = \frac{66 - c_0}{66 - 41} = 0.631 \]

Solving for \( c_0 \) gives \( c_0 = 50.2 \text{ wt}\% \text{ Pb} \).

Now the analogous expression for total \( \alpha \)

\[ W_{\alpha} = \frac{c_{\text{Mg}_2\text{Pb}} - c_0}{c_{\text{Mg}_2\text{Pb}} - c_\alpha} \]
\[ = \frac{81 - c_0}{81 - 41} = 0.896 \]

And this value of \( c_0 \) is 45.2 wt% Pb. Therefore, since these two \( c_0 \) values are different, this alloy is not possible.

9.33 (a) This portion of the problem asks that we determine the mass fractions of \( \alpha \) and \( \beta \) phases for an 25 wt% Ag-75 wt% Cu alloy (at 775°C). In order to do this it is necessary to employ the lever rule using a tie line that extends entirely across the \( \alpha + \beta \) phase field (Figure 9.6), as follows:

\[ W_\alpha = \frac{c_\beta - c_0}{c_\beta - c_\alpha} = \frac{91.2 - 25}{91.2 - 8.0} = 0.796 \]

\[ W_\beta = \frac{c_0 - c_\alpha}{c_\beta - c_\alpha} = \frac{25 - 8.0}{91.2 - 8.0} = 0.204 \]

(b) Now it is necessary to determine the mass fractions of primary \( \alpha \) and eutectic microconstituents for this same alloy. This requires us to utilize the lever rule and a tie line that
extends from the maximum solubility of Ag in the $\alpha$ phase at $775^\circ$C (i.e., 8.0 wt% Ag) to the eutectic composition (71.9 wt% Ag). Thus

$$W_{\alpha} = \frac{C_{\text{eutectic}} - C_{\alpha}}{C_{\text{eutectic}} - C_{\alpha}} = \frac{71.9 - 25}{71.9 - 8.0} = 0.734$$

$$W_{e} = \frac{C_{\alpha} - C_{\text{eutectic}}}{C_{\text{eutectic}} - C_{\alpha}} = \frac{25 - 8.0}{71.9 - 8.0} = 0.266$$

(c) And, finally, we are asked to compute the mass fraction of eutectic $\alpha$, $W_{e\alpha}$. This quantity is simply the difference between the mass fractions of total $\alpha$ and primary $\alpha$ as

$$W_{e\alpha} = W_{\alpha} - W_{\alpha}' = 0.796 - 0.734 = 0.062$$

9.34 This problem asks that we determine the composition of a Pb-Sn alloy at $180^\circ$C given that $W_{\beta}' = 0.57$ and $W_{e} = 0.43$. Since there is a primary $\beta$ microconstituent present, then we know that the alloy composition, $C_{o}$, is between 61.9 and 97.8 wt% Sn (Figure 9.7). Furthermore, this figure also indicates that $C_{\beta} = 97.8$ wt% Sn and $C_{\text{eutectic}} = 61.9$ wt% Sn. Applying the appropriate lever rule expression for $W_{\beta}'$

$$W_{\beta}' = \frac{C_{o} - C_{\text{eutectic}}}{C_{\beta} - C_{\text{eutectic}}} = \frac{C_{o} - 61.9}{97.8 - 61.9} = 0.57$$

and solving for $C_{o}$ yields $C_{o} = 82.4$ wt% Sn.

9.35 We are given a hypothetical eutectic phase diagram for which $C_{\text{eutectic}} = 47$ wt% B, $C_{\beta} = 92.6$ wt% B at the eutectic temperature, and also that $W_{\alpha}' = 0.356$ and $W_{\alpha} = 0.693$; from this we are asked to determine the composition of the alloy. Let us write lever rule expressions for $W_{\alpha}'$ and $W_{\alpha}$

$$W_{\alpha} = \frac{C_{\beta} - C_{o}}{C_{\beta} - C_{\alpha}} = \frac{92.6 - C_{o}}{92.6 - C_{\alpha}} = 0.693$$

$$W_{\alpha}' = \frac{C_{\text{eutectic}} - C_{o}}{C_{\text{eutectic}} - C_{\alpha}} = \frac{47 - C_{o}}{47 - C_{\alpha}} = 0.356$$

Thus, we have two simultaneous equations with $C_{o}$ and $C_{\alpha}$ as unknowns. Solving them for $C_{o}$ gives $C_{o} = 32.6$ wt% B.

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9.36 Upon solidification, an alloy of eutectic composition forms a microstructure consisting of alternating layers of the two solid phases because during the solidification atomic diffusion must occur, and with this layered configuration the diffusion path length for the atoms is a minimum.

9.37 Schematic sketches of the microstructures that would be observed for an 85 wt% Pb-15 wt% Mg alloy at temperatures of 600°C, 500°C, 270°C, and 200°C are shown below. The phase compositions are also indicated.

9.38 Schematic sketches of the microstructures that would be observed for a 68 wt% Zn-32 wt% Cu alloy at temperatures of 1000°C, 760°C, 600°C, and 400°C are shown below. The phase compositions are also indicated.
9.39 Schematic sketches of the microstructures that would be observed for a 30 wt% Zn-70 wt% Cu alloy at temperatures of 1100°C, 950°C, 900°C, and 700°C are shown below. The phase compositions are also indicated.
9.40 The principal difference between congruent and incongruent phase transformations is that for congruent no compositional changes occur with any of the phases that are involved in the transformation. For incongruent there will be compositional alterations of the phases.

9.41 In this problem we are asked to specify temperature-composition points for all eutectics, eutectoids, peritectics, and congruent phase transformations for the aluminum-neodymium system.

There are two eutectics on this phase diagram. One exists at 12 wt% Nd-88 wt% Al and 632\degree C. The reaction upon cooling is

\[ L \rightarrow Al + Al_{11}Nd_3 \]

The other eutectic exists at about 97 wt% Nd-3 wt% Al and 635\degree C. This reaction upon cooling is

\[ L \rightarrow AlNd_3 + Nd \]

There are four peritectics. One exists at 59 wt% Nd-41 wt% Al and 1235\degree C. Its reaction upon cooling is as follows:

\[ \alpha \rightarrow Al + Al_{11}Nd_3 \]
\[ L + Al_2Nd \rightarrow Al_{11}Nd_3 \]

The second peritectic exists at 84 wt% Nd-16 wt% Al and 940°C. This reaction upon cooling is

\[ L + Al_2Nd \rightarrow AlNd \]

The third peritectic exists at 91 wt% Nd-9 wt% Al and 795°C. This reaction upon cooling is

\[ L + AlNd \rightarrow AlNd_2 \]

The fourth peritectic exists at 94 wt% Nd-6 wt% Al and 675°C. This reaction upon cooling is

\[ L + AlNd_2 \rightarrow AlNd_3 \]

There is one congruent melting point at about 73 wt% Nd-27 wt% Al and 1460°C. Its reaction upon cooling is

\[ L \rightarrow Al_2Nd \]

No eutectoids are present.

9.42 In this problem we are asked to specify temperature-composition points for all eutectics, eutectoids, peritectics, and congruent phase transformations for a portion of the titanium-copper phase diagram.

There is one eutectic on this phase diagram, which exists at about 51 wt% Cu-49 wt% Ti and 960°C. Its reaction upon cooling is

\[ L \rightarrow Ti_2Cu + TiCu \]

There is one eutectoid for this system. It exists at about 7.5 wt% Cu-92.5 wt% Ti and 790°C. This reaction upon cooling is

\[ \beta \rightarrow \alpha + Ti_2Cu \]

There is one peritectic on this phase diagram. It exists at about 40 wt% Cu-60 wt% Ti and 1005°C. The reaction upon cooling is
There is a single congruent melting point that exists at about 57.5 wt% Cu-42.5 wt% Ti and 982°C. The reaction upon cooling is

\[ \beta + L \rightarrow Ti_2Cu \]

9.43 This problem asks for us to compute the maximum number of phases that may be present for a ternary system assuming that pressure is held constant. For a ternary system (\(C = 3\)) at constant pressure (\(N = 1\)), Gibbs phase rule, Equation (9.16), becomes

\[ P + F = C + N = 3 + 1 = 4 \]

Or,

\[ P = 4 - F \]

Thus, when \(F = 0\), \(P\) will have its maximum value of 4, which means that the maximum number of phases present for this situation is 4.

9.44 We are asked to specify the value of \(F\) for Gibbs phase rule at points A, B, and C on the pressure-temperature diagram for H₂O. Gibbs phase rule in general form is

\[ P + F = C + N \]

For this system, the number of components, \(C\), is 1, whereas \(N\), the number of noncompositional variables, is 2—viz. temperature and pressure. Thus, the phase rule now becomes

\[ P + F = 1 + 2 = 3 \]

Or

\[ F = 3 - P \]

where \(P\) is the number of phases present at equilibrium.

At point A, three phases are present (viz. ice I, ice III, and liquid) and \(P = 3\); thus, the number of degrees of freedom is zero since
\[ F = 3 - P = 3 - 3 = 0 \]

Thus, point A is an invariant point (in this case a triple point), and we have no choice in the selection of externally controllable variables in order to define the system.

At point B on the figure, only a single (vapor) phase is present (i.e., \( P = 1 \)), or

\[ F = 3 - P = 3 - 1 = 2 \]

which means that both temperature and pressure are necessary to define the system.

And, finally, at point C which is on the phase boundary between liquid and ice I phases, two phases are in equilibrium (\( P = 2 \)); hence

\[ F = 3 - P = 3 - 2 = 1 \]

Or that we need to specify the value of either temperature or pressure, which determines the value of the other (pressure or temperature).

9.45 Below is shown the phase diagram for these two A and B metals.
This problem gives us the compositions in weight percent for the two intermetallic compounds AB and AB\(_2\), and then asks us to identify element B if element A is potassium. Probably the easiest way to solve this problem is to first compute the ratio of the atomic weights of these two elements using Equation (4.6a); then, since we know the atomic weight of potassium (39.10 g/mol), it is possible to determine the atomic weight of element B, from which an identification may be made.

First of all, consider the AB intermetallic compound; inasmuch as it contains the same numbers of A and B atoms, its composition in atomic percent is 50 at% A-50 at% B. Equation (4.6a) may be written in the form:

\[
C_B^* = \frac{C_B^A A_B}{C_A^A A_B + C_B^B A_A} \times 100
\]

where \(A_A\) and \(A_B\) are the atomic weights for elements A and B, and \(C_A\) and \(C_B\) are their compositions in weight percent. For this AB compound, and making the appropriate substitutions in the above equation leads to
50 at% B = \frac{(65.7 \text{ wt% B})(A_A)}{(34.3 \text{ wt% A})(A_B) + (65.7 \text{ wt% B})(A_A)} \times 100

Now, solving this expression yields,

\[ A_B = 1.916 \, A_A \]

Since potassium is element A and it has an atomic weight of 39.10 g/mol, the atomic weight of element B is just

\[ A_B = (1.916)(39.10 \text{ g/mol}) = 74.92 \text{ g/mol} \]

Upon consultation of the period table of the elements (Figure 2.6) we note that arsenic has an atomic weight of 74.92 g/mol; therefore, element B is arsenic.

9.47 This problem asks that we compute the mass fractions of ferrite and cementite in pearlite. The lever-rule expression for ferrite is

\[ W_\alpha = \frac{C_{Fe_3C} \cdot C_o}{C_{Fe_3C} \cdot C_\alpha} \]

and, since \( C_{Fe_3C} = 6.70 \text{ wt}\% \text{ C} \), \( C_o = 0.76 \text{ wt}\% \text{ C} \), and \( C_\alpha = 0.022 \text{ wt}\% \text{ C} \)

\[ W_\alpha = \frac{6.70 \cdot 0.76}{6.70 - 0.022} = 0.89 \]

Similarly, for cementite

\[ W_{Fe_3C} = \frac{C_o \cdot C_\alpha}{C_{Fe_3C} \cdot C_\alpha} = \frac{0.76 \cdot 0.022}{6.70 - 0.022} = 0.11 \]

9.48 A phase is a homogeneous portion of the system having uniform physical and chemical characteristics, whereas a microconstituent is an identifiable element of the microstructure (that may consist of more than one phase).
9.49 (a) A **hypoeutectoid** steel has a carbon concentration less than the eutectoid; on the other hand, a **hypereutectoid** steel has a carbon content greater than the eutectoid.

(b) For a hypoeutectoid steel, the proeutectoid ferrite is a microconstituent that formed above the eutectoid temperature. The eutectoid ferrite is one of the constituents of pearlite that formed at a temperature below the eutectoid. The carbon concentration for both ferrites is 0.022 wt% C.

9.50 A proeutectoid phase normally forms along austenite grain boundaries because there is an interfacial energy associated with these boundaries. When a proeutectoid phase forms within austenite, an interfacial energy also exists at the interface between the two phases. A lower net interfacial energy increase results when the proeutectoid phase forms along the existing austenite grain boundaries than when the proeutectoid phase forms within the interior of the grains.

9.51 This problem asks that we compute the carbon concentration of an iron-carbon alloy for which the fraction of total ferrite is 0.94. Application of the lever rule [of the form of Equation (9.12)] yields

\[
W_\alpha = 0.94 = \frac{C_{Fe_3C} - C'_o}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - C'_o}{6.70 - 0.022}
\]

and solving for \(C'_o\)

\[C'_o = 0.42\ wt\%\ C\]

9.52 In this problem we are given values of \(W_\alpha\) and \(W_{Fe_3C}\) for an iron-carbon alloy and then are asked to specify the proeutectoid phase. Employment of the lever rule for total \(\alpha\) leads to

\[
W_\alpha = 0.92 = \frac{C_{Fe_3C} - C_o}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - C_o}{6.70 - 0.022}
\]

Now, solving for \(C_o\), the alloy composition, leads to \(C_o = 0.56\ wt\%\ C\). Therefore, the proeutectoid phase is \(\alpha\)-ferrite since \(C_o\) is less than 0.76 wt% C.
This problem asks us to consider various aspects of 1.0 kg of austenite containing 1.15 wt% C that is cooled to below the eutectoid.

(a) The proeutectoid phase will be Fe₃C since 1.15 wt% C is greater than the eutectoid (0.76 wt% C).

(b) For this portion of the problem, we are asked to determine how much total ferrite and cementite form. Application of the appropriate lever rule expression yields

\[ W_\alpha = \frac{C_{Fe_3C} - C_\alpha}{C_{Fe_3C} - C_o} = \frac{6.70 - 1.15}{6.70 - 0.022} = 0.83 \]

which, when multiplied by the total mass of the alloy (1.0 kg), gives 0.83 kg of total ferrite.

Similarly, for total cementite,

\[ W_{Fe_3C} = \frac{C_o - C_\alpha}{C_{Fe_3C} - C_\alpha} = \frac{1.15 - 0.022}{6.70 - 0.022} = 0.17 \]

And the mass of total cementite that forms is (0.17)(1.0 kg) = 0.17 kg.

(c) Now we are asked to calculate how much pearlite and the proeutectoid phase (cementite) form. Applying Equation (9.22), in which \( C_1' = 1.15 \) wt% C

\[ W_p = \frac{6.70 - C_1'}{6.70 - 0.76} = \frac{6.70 - 1.15}{6.70 - 0.76} = 0.93 \]

which corresponds to a mass of 0.93 kg. Likewise, from Equation (9.23)

\[ W_{Fe_3C'} = \frac{C_1' - 0.76}{5.94} = \frac{1.15 - 0.76}{5.94} = 0.07 \]

which is equivalent to 0.07 kg of the total 1 kg mass.

(d) Schematically, the microstructure would appear as:
We are called upon to consider various aspects of 2.5 kg of austenite containing 0.65 wt% C, that is cooled to below the eutectoid.

(a) Ferrite is the proeutectoid phase since 0.65 wt% C is less than 0.76 wt% C.

(b) For this portion of the problem, we are asked to determine how much total ferrite and cementite form. Application of the appropriate lever rule expression yields

\[
W_\alpha = \frac{C_{Fe_3C} - C_\alpha}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - 0.65}{6.70 - 0.022} = 0.91
\]

which corresponds to \((0.91)(2.5 \text{ kg}) = 2.26 \text{ kg}\) of total ferrite.

Similarly, for total cementite,

\[
W_{Fe_3C} = \frac{C_\alpha - C_{Fe_3C}}{C_{Fe_3C} - C_\alpha} = \frac{0.65 - 0.022}{6.70 - 0.022} = 0.09
\]

Or \((0.09)(2.5 \text{ kg}) = 0.24 \text{ kg}\) of total cementite form.

(c) Now consider the amounts of pearlite and proeutectoid ferrite. Using Equation (9.20)

\[
W_p = \frac{C_\alpha' - 0.022}{0.74} = \frac{0.65 - 0.022}{0.74} = 0.85
\]

This corresponds to \((0.85)(2.5 \text{ kg}) = 2.12 \text{ kg}\) of pearlite.

Also, from Equation (9.21),

\[
W_{\alpha'} = \frac{0.76 - 0.65}{0.74} = 0.15
\]
Or, there are \((0.15)(2.5 \text{ kg}) = 0.38 \text{ kg}\) of proeutectoid ferrite.

(d) Schematically, the microstructure would appear as:

9.55 The mass fractions of proeutectoid ferrite and pearlite that form in a 0.25 wt% C iron-carbon alloy are considered in this problem. From Equation (9.20)

\[
W_p = \frac{C'_o - 0.022}{0.74} = \frac{0.25 - 0.022}{0.74} = 0.31
\]

And, from Equation (9.21)

\[
W_{\alpha'} = \frac{0.76 - C'_o}{0.74} = \frac{0.76 - 0.25}{0.74} = 0.69
\]

9.56 This problem asks that we determine the carbon concentration in an iron-carbon alloy, given the mass fractions of proeutectoid ferrite and pearlite. From Equation (9.20)

\[
W_p = 0.714 = \frac{C'_o - 0.022}{0.74}
\]

which yields \(C'_o = 0.55 \text{ wt% C}\).

9.57 In this problem we are given values of \(W_{\alpha'}\) and \(W_{Fe_3C}\) for an iron-carbon alloy (0.88 and 0.12, respectively) and then are asked to specify whether the alloy is hypoeutectoid or hypereutectoid. Employment of the lever rule for total \(\alpha\) leads to
Now, solving for \( C_o \), the alloy composition, leads to \( C_o = 0.82 \) wt% C. Therefore, the alloy is hypereutectoid since \( C_o \) is greater than 0.76 wt% C.

9.58 We are asked in this problem to determine the concentration of carbon in an alloy for which \( W_{\alpha'} = 0.20 \) and \( W_p = 0.80 \). If we let \( C_o \) equal the carbon concentration in the alloy, employment of the appropriate lever rule expression, Equation (9.20), leads to

\[
W_p = \frac{C_o - 0.022}{0.76 - 0.022} = 0.80
\]

Solving for \( C_o \) yields \( C_o = 0.61 \) wt% C.

9.59 In this problem we are asked to consider 2.0 kg of a 99.6 wt% Fe-0.4 wt% C alloy that is cooled to a temperature below the eutectoid.

(a) Equation (9.21) must be used in computing the amount of proeutectoid ferrite that forms. Thus,

\[
W_{\alpha'} = \frac{0.76 - C_o'}{0.74} = \frac{0.76 - 0.40}{0.74} = 0.49
\]

Or, \((0.49)(2.0 \text{ kg}) = 0.99 \text{ kg}\) of proeutectoid ferrite forms.

(b) In order to determine the amount of eutectoid ferrite, it first becomes necessary to compute the amount of total ferrite using the lever rule applied entirely across the \( \alpha + \text{Fe}_3\text{C} \) phase field, as

\[
W_\alpha = \frac{C_{\text{Fe}_3\text{C}} - C_o}{C_{\text{Fe}_3\text{C}} - C_\alpha} = \frac{6.70 - 0.40}{6.70 - 0.022} = 0.94
\]

which corresponds to \((0.94)(2.0 \text{ kg}) = 1.89 \text{ kg}\). Now, the amount of eutectoid ferrite is just the difference between total and proeutectoid ferrites, or

\[
1.89 \text{ kg} - 0.99 \text{ kg} = 0.90 \text{ kg}
\]
(c) With regard to the amount of cementite that forms, again application of the lever rule across the entirety of the $\alpha + \text{Fe}_3\text{C}$ phase field, leads to

$$w_{\text{Fe}_3\text{C}} = \frac{C'_1 - C_{\alpha}}{C_{\text{Fe}_3\text{C}} - C_{\alpha}} = \frac{0.40 - 0.022}{6.70 - 0.022} = 0.06$$

which amounts to $(0.06)(2 \text{ kg}) = 0.11 \text{ kg}$ cementite in the alloy.

9.60 This problem asks that we compute the maximum mass fraction of proeutectoid cementite possible for a hypereutectoid iron-carbon alloy. This requires that we utilize Equation (9.23) with $C'_1 = 2.14 \text{ wt\% C}$, the maximum solubility of carbon in austenite. Thus,

$$w_{\text{Fe}_3\text{C}}' = \frac{C'_1 - 0.76}{5.94} = \frac{2.14 - 0.76}{5.94} = 0.232$$

9.61 This problem asks if it is possible to have an iron-carbon alloy for which $w_{\alpha} = 0.846$ and $w_{\text{Fe}_3\text{C}}' = 0.049$. In order to make this determination, it is necessary to set up lever rule expressions for these two mass fractions in terms of the alloy composition, then to solve for the alloy composition of each; if both alloy composition values are equal, then such an alloy is possible. The expression for the mass fraction of total ferrite is

$$w_{\alpha} = \frac{C_{\text{Fe}_3\text{C}} - C_0}{C_{\text{Fe}_3\text{C}} - C_{\alpha}} = \frac{6.70 - C_0}{6.70 - 0.022} = 0.846$$

Solving for this $C_0$ yields $C_0 = 1.05 \text{ wt\% C}$. Now for $w_{\text{Fe}_3\text{C}}'$ we utilize Equation (9.23) as

$$w_{\text{Fe}_3\text{C}}' = \frac{C'_1 - 0.76}{5.94} = 0.049$$

This expression leads to $C'_1 = 1.05 \text{ wt\% C}$. And, since $C_0 = C'_1$, this alloy is possible.

9.62 This problem asks if it is possible to have an iron-carbon alloy for which $w_{\text{Fe}_3\text{C}} = 0.039$ and $w_p = 0.417$. In order to make this determination, it is necessary to set up lever rule expressions for these two mass fractions in terms of the alloy composition, then to solve for the alloy composition of each; if both alloy composition values are equal, then such an alloy is possible. The expression for the mass fraction of total cementite is
\[ W_{\text{Fe}_3\text{C}} = \frac{C_o \cdot C_\alpha}{C_{\text{Fe}_3\text{C}} \cdot C_\alpha} = \frac{C_o \cdot 0.022}{6.70 - 0.022} = 0.039 \]

Solving for this \( C_o \) yields \( C_o = 0.28 \text{ wt}\% \text{ C} \). Now for \( W_p \) we utilize Equation (9.20) as

\[ W_p = \frac{C_o' - 0.022}{0.74} = 0.417 \]

This expression leads to \( C_o' = 0.33 \text{ wt}\% \text{ C} \). Since \( C_o \) and \( C_o' \) are different, this alloy is not possible.

9.63 This problem asks that we compute the mass fraction of eutectoid ferrite in an iron-carbon alloy that contains 0.43 wt\% C. In order to solve this problem it is necessary to compute mass fractions of total and proeutectoid ferrites, and then to subtract the latter from the former. To calculate the mass fraction of total ferrite, it is necessary to use the lever rule and a tie line that extends across the entire \( \alpha + \text{Fe}_3\text{C} \) phase field as

\[ W_\alpha = \frac{C_{\text{Fe}_3\text{C}} \cdot C_o}{C_{\text{Fe}_3\text{C}} \cdot C_\alpha} = \frac{6.70 - 0.43}{6.70 - 0.022} = 0.939 \]

Now, for the mass fraction of proeutectoid ferrite we use Equation (9.21)

\[ W_{\alpha'} = \frac{0.76 \cdot C_o'}{0.74} = \frac{0.76 \cdot 0.43}{0.74} = 0.446 \]

And, finally, the mass fraction of eutectoid ferrite \( W_{\alpha''} \) is just

\[ W_{\alpha''} = W_\alpha - W_{\alpha'} = 0.939 - 0.446 = 0.493 \]

9.64 This problem asks whether or not it is possible to determine the composition of an iron-carbon alloy for which the mass fraction of eutectoid cementite is 0.104; and if so, to calculate the composition. Yes, it is possible to determine the alloy composition; and, in fact, there are two possible answers. For the first, the eutectoid cementite exists in addition to proeutectoid cementite. For this case the mass fraction of eutectoid cementite \( (W_{\text{Fe}_3\text{C}''}) \) is just the difference between total cementite and proeutectoid cementite mass fractions; that is
\[ W_{Fe_3C''} = W_{Fe_3C} - W_{Fe_3C'} \]

Now, it is possible to write expressions for \( W_{Fe_3C} \) and \( W_{Fe_3C'} \) in terms of \( C_o \), the alloy composition. Thus,

\[
W_{Fe_3C''} = \frac{C_o - C_\alpha}{C_{Fe_3C} - C_\alpha} \cdot \frac{C_o - 0.76}{5.93} \\
= \frac{C_o - 0.022}{6.70 - 0.022} \cdot \frac{C_o - 0.76}{5.94} = 0.104
\]

And, solving for \( C_o \) yields \( C_o = 1.11 \) wt\% C.

For the second possibility, we have a hypoeutectoid alloy wherein all of the cementite is eutectoid cementite. Thus, it is necessary to set up a lever rule expression wherein the mass fraction of total cementite is 0.104. Therefore,

\[
W_{Fe_3C} = \frac{C_o - C_\alpha}{C_{Fe_3C} - C_\alpha} = \frac{C_o - 0.022}{6.70 - 0.022} = 0.104
\]

And, solving for \( C_o \) yields \( C_o = 0.72 \) wt\% C.

9.65 This problem asks whether or not it is possible to determine the composition of an iron-carbon alloy for which the mass fraction of eutectoid ferrite is 0.82; and if so, to calculate the composition. Yes, it is possible to determine the alloy composition; and, in fact, there are two possible answers. For the first, the eutectoid ferrite exists in addition to proeutectoid ferrite. For this case the mass fraction of eutectoid ferrite \( (W_{\alpha''}) \) is just the difference between total ferrite and proeutectoid ferrite mass fractions; that is

\[ W_{\alpha''} = W_\alpha - W_\alpha'. \]

Now, it is possible to write expressions for \( W_\alpha \) and \( W_\alpha' \), in terms of \( C_o \), the alloy composition. Thus,

\[
W_{\alpha''} = \frac{C_{Fe_3C} - C_o}{C_{Fe_3C} - C_\alpha} \cdot \frac{0.76 - C_o}{0.74}
\]
\[
= \frac{6.70 \cdot C_o - 0.022}{6.70 - C_o} \cdot \frac{0.76 \cdot C_o}{0.74} = 0.82
\]

And, solving for \( C_o \) yields \( C_o = 0.70 \) wt% C.

For the second possibility, we have a hypereutectoid alloy wherein all of the ferrite is eutectoid ferrite. Thus, it is necessary to set up a lever rule expression wherein the mass fraction of total ferrite is 0.82. Therefore,

\[
W_\alpha = \frac{C_{Fe_3C} - C_o}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - C_o}{6.70 - 0.022} = 0.82
\]

And, solving for \( C_o \) yields \( C_o = 1.22 \) wt% C.

9.66 Schematic microstructures for the iron-carbon alloy of composition 5 wt% C-95 wt% Fe and at temperatures of 1175°C, 1145°C, and 700°C are shown below; approximate phase compositions are also indicated.
9.67  This problem asks that we determine the approximate Brinell hardness of a 99.8 wt% Fe-0.2 wt% C alloy. First, we compute the mass fractions of pearlite and proeutectoid ferrite using Equations (9.20) and (9.21), as

\[
W_p = \frac{C_0' - 0.022}{0.74} = \frac{0.20 - 0.022}{0.74} = 0.24
\]

\[
W_{\alpha'} = \frac{0.76 - C_0'}{0.74} = \frac{0.76 - 0.20}{0.74} = 0.76
\]

Now, we compute the Brinell hardness of the alloy as

\[
HB_{\text{alloy}} = HB_{\alpha'} W_{\alpha'} + HB_p W_p
\]

\[= (80)(0.76) + (280)(0.24) = 128\]

9.68  We are asked in this problem to estimate the composition of the Pb-Sn alloy which microstructure is shown in Figure 9.15. Primary \(\alpha\) and eutectic microconstituents are present in the photomicrograph, and it is given that their densities are 11.2 and 8.7 g/cm\(^3\), respectively. Below is shown a square grid network onto which is superimposed outlines of the primary \(\alpha\) phase areas.

The area fraction of this primary \(\alpha\) phase may be determined by counting squares. There are a total of 644 squares, and of these, approximately 104 lie within the primary \(\alpha\) phase particles. Thus, the area fraction of primary \(\alpha\) is 104/644 = 0.16, which is also assumed to be the volume fraction.
We now want to convert the volume fractions into mass fractions in order to employ the lever rule to the Pb-Sn phase diagram. To do this, it is necessary to utilize Equations (9.7a) and (9.7b) as follows:

\[
W_{\alpha'} = \frac{V_{\alpha'} \rho_{\alpha'}}{V_{\alpha'} \rho_{\alpha'} + V_{\text{eutectic}} \rho_{\text{eutectic}}}
\]

\[
= \frac{(0.16)(11.2 \text{ g/cm}^3)}{(0.16)(11.2 \text{ g/cm}^3) + (0.84)(8.7 \text{ g/cm}^3)} = 0.197
\]

\[
W_{\text{eutectic}} = \frac{V_{\text{eutectic}} \rho_{\text{eutectic}}}{V_{\alpha'} \rho_{\alpha'} + V_{\text{eutectic}} \rho_{\text{eutectic}}}
\]

\[
= \frac{(0.84)(8.7 \text{ g/cm}^3)}{(0.16)(11.2 \text{ g/cm}^3) + (0.84)(8.7 \text{ g/cm}^3)} = 0.803
\]

From Figure 9.7, we want to use the lever rule and a tie-line that extends from the eutectic composition (61.9 wt% Sn) to the \(\alpha-(\alpha + \beta)\) phase boundary at 180°C (about 18.3 wt% Sn). Accordingly

\[
W_{\alpha'} = 0.197 = \frac{61.9 - C_o}{61.9 - 18.3}
\]

wherein \(C_o\) is the alloy composition (in wt% Sn). Solving for \(C_o\) yields \(C_o = 53.3\) wt% Sn.

9.69 This problem asks us to consider an alloy of composition 97.5 wt% Fe, 2.0 wt% Mo, and 0.5 wt% C.

(a) From Figure 9.32, the eutectoid temperature for 2.0 wt% Mo is approximately 850°C.

(b) From Figure 9.33, the eutectoid composition is approximately 0.22 wt% C.

(c) Since the carbon concentration of the alloy (0.5 wt%) is greater than the eutectoid, cementite is the proeutectoid phase.

9.70 We are asked to consider a steel alloy of composition 93.8 wt% Fe, 6.0 wt% Ni, and 0.2 wt% C.

(a) From Figure 9.32, the eutectoid temperature for 6 wt% Ni is approximately 650°C (1200°F).

(b) From Figure 9.33, the eutectoid composition is approximately 0.62 wt% C. Since the carbon concentration in the alloy (0.2 wt%) is less than the eutectoid, the proeutectoid phase is ferrite.
(c) Assume that the $\alpha-(\alpha + Fe_3C)$ phase boundary is at a negligible carbon concentration. Modifying Equation (9.21) leads to

$$W_\alpha = \frac{0.62 - C_0'}{0.62 - 0} = \frac{0.62 \cdot 0.20}{0.62} = 0.68$$

Likewise, using a modified Equation (9.20)

$$W_p = \frac{C_0' - 0}{0.62 - 0} = \frac{0.20}{0.62} = 0.32$$